

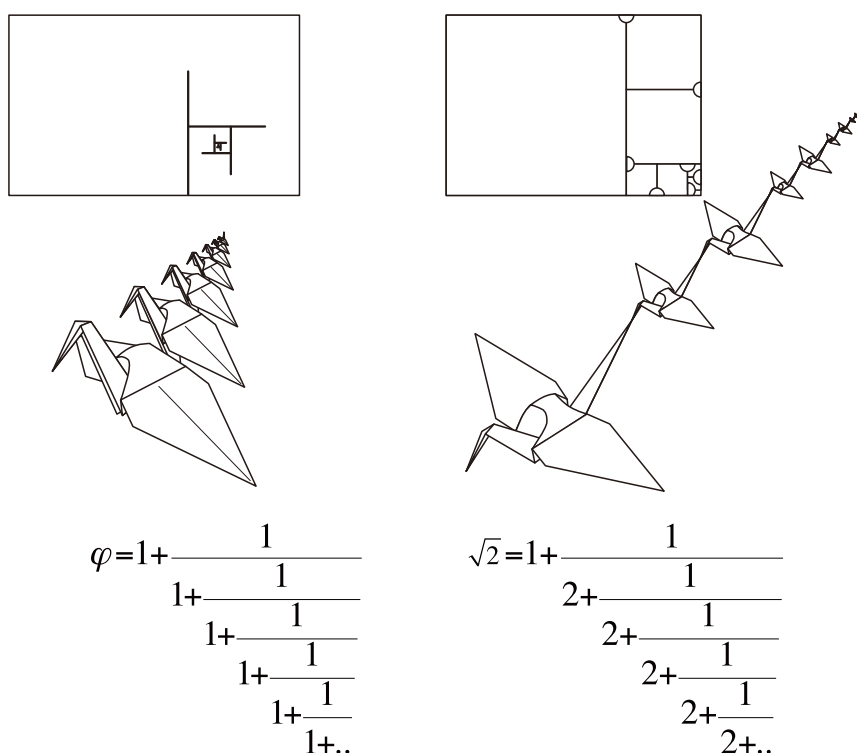
# Designing Connected Paper Cranes by Using Continued Fractions

*J. Maekawa*

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## Abstract

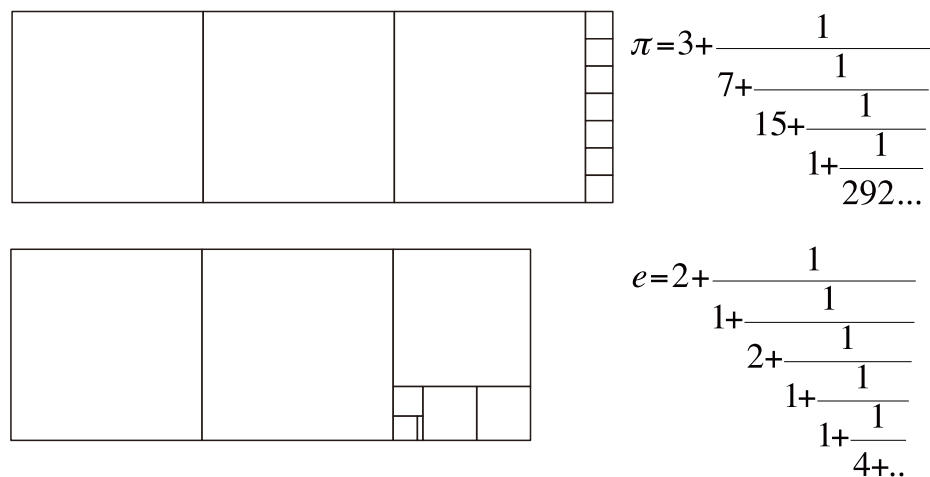
Connected paper cranes, represented by "Hiden Sembazuru Orikata" (1797, R. Akisato), constitute one genre of traditional origami designs. The author advanced the technique to have designed two pieces of connected cranes that have infinitely repeating structures. One was made from the golden rectangle, the other from the  $1:\sqrt{2}$  rectangle (sometimes called the silver rectangle). If you were to remove squares from these rectangles, you would repeat same procedures infinitely. Such property of the rectangles can be expressed by continued fractions. You can remove one square from a rectangle if the corresponding partial denominator of the regular continued fraction (whose partial numerators are always 1) is 1, and can remove two if the partial denominator is 2. That is illustrated in Figure 1.



**Figure 1:** Connected cranes from the golden and "silver" rectangles (2007, 2009)

These designs intuitively represent the golden ratio and square root of two to be irrational numbers, i.e. numbers that cannot be expressed by a ratio of integers.

Beside the golden ratio and square root of two shown in Figure 1, any irrational numbers can be expressed by continued fractions. For example,  $\pi$  (the ratio of the circumference of a circle to its diameter) and  $e$  (the base of natural logarithms) are expressed as shown in Figure 2. These fractions can also be applied to connected crane designs.



**Figure 2:** Continued fractions expressing  $\pi$  and  $e$

In this study, the author shows various examples of connected cranes based on continued fractions. The idea is then extended beyond regular continued fractions, in which the partial numerators are always 1, to generalized continued fractions. The author also shows connected crane examples that are applications of geometric series or harmonic series.

Such art works visualize harmony found in Mathematics.